Epistemic Queries for OWL Knowledge Bases Technical Report

Anees ul Mehdi, Sebastian Rudolph, and Stephan Grimm

Institute AIFB, Karlsruhe Institute of Technology, DE
{sebastian.rudolph,anees.mehdi}@kit.edu
Forschungszentrum Informatik Karlsruhe, DE
grimm@fzi.de

Abstract. Extensions of Description Logics (DLs) with certain modalities have been addressed in several works. Algorithms have also been designed for the reasoning services in such extensions. In this work, we focus on DLs extended only by the modal operator K called "epistemic operator". We propose a technique for answering queries of such extensions when put to a standard DL knowledge base, that is, we restrict the language for representing a knowledge base to standard DLs and allow a richer language (standard DL extended by K) for querying such knowledge bases. The main contribution of our work is a novel technique that translates epistemic queries into standard DL queries. This allows for using standard DL reasoners for answering epistemic queries.

1 Introduction

Description logics (DLs) are a family of logic-based knowledge representation formalisms providing more expressivity than propositional logic and allowing for decidable reasoning services [**?**]. Among several, *querying* a DL-knowledge base is one of the most important reasoning services. Extensive work has been done in this direction presenting not only complexity results for such services but also extending them from simple forms of querying (*instance retrieval*) to more complex forms (*conjunctive query answering*), see e.g., [**?**] for more details.

In the early 80s Hector J. Levesque argued for the need for a richer query language in knowledge formalisms [**?**]. He describes that the approach to knowledge representation should be functional rather than structural and defends the idea of extending a querying language by the attribute *knows* denoted by **K** (a modality in Modal Logic terminology). In [**?**], Raymond Reiter makes a similar argument of in-adequacy of the standard first-order language for querying. Nevertheless, he discusses this issue in the context of databases. Similar lines of argumentation can be seen in the DL-community as well [?,?,?,?] where several extensions of DLs have been presented as well as algorithms for deciding the reasoning services in such extensions. The extension of the DL $\mathcal{ALC}$ [?] by the epistemic operator $\mathbf{K}$ called $\mathcal{ALCK}$, is presented in [?]. A tableau algorithm has been designed for deciding the satisfiability problem. Answering queries in $\mathcal{ALCK}$ put to $\mathcal{ALC}$ knowledge bases is also discussed. In this work we mainly focus on DLs extended with the epistemic operator $\mathbf{K}$ following notions presented in [?]. However, we consider more expressive DLs rather than just $\mathcal{ALC}$.

The rest of this paper is structured as follows. In Section 2, we start by providing a formal syntax and semantics of the description logic $\mathcal{SROIQ}$ which is an extension of the DL $\mathcal{ALC}$ by several constructs. We then present the extension of $\mathcal{SROIQ}$ with the epistemic operator $\mathbf{K}$. We call this extension $\mathcal{SROIQK}$. We also introduce different reasoning tasks in $\mathcal{SROIQ}$ and $\mathcal{SROIQK}$. In Section 3, we present the notion of using $\mathcal{SROIQK}$ for *querying a knowledge base* and present our method of answering such queries. First we consider the case where we restrict the language of the input knowledge base to $\mathcal{SRIQ}$. We start by providing an example and later on formalize the technique by providing a translation function which reduces such $\mathcal{SROIQK}$ queries to standard DL queries i.e., queries without involving the $\mathbf{K}$-operator. We also provide the proof of correctness of our method in the same section. In Section 4, we will see that what problems we encounter when allowing for $\mathcal{SROIQ}$ as the knowledge base language. In Section 5, we discuss the implementation issues and some evaluation results. Finally the concluding remarks and related work are finally presented in Section 6.

2 Preliminaries

In this section, we present an introduction to the description logic $\mathcal{SROIQ}$ and its extension with the epistemic operator $\mathbf{K}$.

2.1 Description Logics $\mathcal{SROIQ}$

We start by presenting the syntax and semantics of $\mathcal{SROIQ}$. It is an extension of $\mathcal{ALC}$ with inverse roles($\mathcal{I}$), role hierarchies($\mathcal{H}$), nominals($\mathcal{O}$) and qualifying number restrictions($\mathcal{Q}$). Besides it also allows for several role constructs and axioms.

Definition 1. For the signature of $\mathcal{SROIQ}$ we have finite and disjoint sets N_C, N_R and N_I of *concept names*, *role names* and *individual names* respectively.[1] Further the set N_R is partitioned into two sets namely, $\mathbf{R_s}$ and $\mathbf{R_n}$ of *simple* and *non-simple* roles respectively. The set $\mathbf{R}$ *of* $\mathcal{SROIQ}$*-roles* is

$$\mathbf{R} := U \mid N_R \mid N_R^-$$

where U is called the *universal role*. Further, we define a function Inv on roles such that $\mathsf{Inv}(R) = R^-$ if R is a role name, $\mathsf{Inv}(R) = S$ if $R = S^-$ and $\mathsf{Inv}(U) := U$.

The set of $\mathcal{SROIQ}$-concepts is the smallest set satisfying the following properties:

- every concept name $A \in N_C$ is a concept;
- $\top$(top concept) and $\bot$ (bottom concept) are concept;
- if C, D are concepts, R is a role, S is a simple role, $a_1, \ldots, a_n$ are individual names and n a non-negative integer then following are concepts:

$$\begin{array}{rl} \neg C & \text{(negation)} \\ \exists S.\mathsf{Self} & \text{(self)} \\ C \sqcap D & \text{(conjunction)} \\ C \sqcup D & \text{(disjunction)} \\ \forall R.C & \text{(universal quantification)} \\ \exists R.C & \text{(existential quantification)} \\ \leq nS.C & \text{(at least number restriction)} \\ \geq nS.C & \text{(at most number restriction)} \\ \{a_1, \ldots, a_n\} & \text{(nominals / one-of)} \end{array}$$

An *RBox axiom* is an expression of one the following forms:

1. $R_1 \circ \cdots \circ R_n \sqsubseteq R$ where $R_1, \ldots, R_n, R \in \mathbf{R}$ and if $n = 1$ and $R_1 \in \mathbf{R_s}$ then $R \notin \mathbf{R_n}$,
2. $\mathsf{Ref}(R)$ (reflexivity), $\mathsf{Tra}(R)$ (transitivity), $\mathsf{Irr}(R)$ (irreflexivity), $\mathsf{Dis}(R, R')$ (role disjointness), $\mathsf{Sym}(R)$ (Symmetry), $\mathsf{Asy}(R)$ (Asymmetry) with $R, R \in \mathbf{R}$.

RBox axioms of the first form i.e., $R_1 \circ \cdots \circ R_n \sqsubseteq R$ are called *role inclusion axioms* (RIAs). An RIA is *complex* if $n > 1$. Whereas the RBox axioms of the second form e.g., $\mathsf{Ref}(R)$, are called *role characteristics*. A

[1] Finiteness, in particular for N_I, is required for the further considerations. However note that the signature is not bounded and can be extended whenever this should be necessary.

$\mathcal{SROIQ}$-RBox $\mathcal{R}$ is a finite set of RBox axioms such that the following conditions are satisfied:[2]

– there is a strict (irreflexive) total order $\prec$ on $\mathbf{R}$ such that
  - for $R \in \{S, \mathsf{Inv}(S)\}$, we have that $S \prec R$ iff $\mathsf{Inv}(S) \prec R$ and
  - every RIA is of the form $R \circ R \sqsubseteq R$, $\mathsf{Inv(R)} \sqsubseteq R$, $R_1 \circ \ldots R_n \sqsubseteq R$, $R \circ R_1 \circ \cdots \circ R_n \sqsubseteq R$ or $R_1 \circ \cdots \circ R_n \circ R \sqsubseteq R$ where $R, R_1 \ldots, R_n \in \mathbf{R}$ and $R_i \prec R$ for $1 \leq i \leq n$.

– any role characteristic of the form $\mathsf{Irr}(S)$, $\mathsf{Dis}(S, S')$ or $\mathsf{Asy}(S)$ is such that $S, S' \in \mathbf{R_s}$ i.e., we allow only for simple role in these role characteristics.

A $\mathcal{SROIQ}$ *general concept inclusion axiom* (GCI) is an expression of the form $C \sqsubseteq D$, where C and D are $\mathcal{SROIQ}$-concepts. A $\mathcal{SROIQ}$*-TBox* is a finite set of $\mathcal{SROIQ}$-GCIs.

An $\mathcal{SROIQ}$*-ABox axiom* is of the form $C(a)$, $R(a, b)$, $a \doteq b$ or $a \neq b$ for the individual names a and b, $\mathcal{SROIQ}$-role R and a $\mathcal{SROIQ}$-concept C. A $\mathcal{SROIQ}$*-ABox* is a finite set of $\mathcal{SROIQ}$-ABox axioms.

A $\mathcal{SROIQ}$*-knowledge base* is a tuple $(\mathcal{T}, \mathcal{R}, \mathcal{A})$ where $\mathcal{T}$ is a $\mathcal{SROIQ}$-TBox, $\mathcal{SROIQ}$-$\mathcal{R}$ is a role hierarchy and $\mathcal{SROIQ}$-$\mathcal{A}$ is a ABox. $\Diamond$

To define the semantics of $\mathcal{SROIQ}$, we introduce the notion of interpretations.

Definition 2. A $\mathcal{SROIQ}$-interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ is composed of a non-empty set $\Delta^{\mathcal{I}}$ called the *domain of* $\mathcal{I}$ and a *mapping function* $\cdot^{\mathcal{I}}$ such that:

– $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ for every concept name A;
– $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ for every $R \in N_R$;
– $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ for every individual name a.

Further the universal role U is interpreted as a total relation on $\Delta^{\mathcal{I}}$ i.e., $U^{\mathcal{I}} = \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. The bottom concept $\bot$ and top concept $\top$ are interpreted by $\emptyset$ and $\Delta^{\mathcal{I}}$ respectively. Now the mapping $.^{\mathcal{I}}$ is extended to roles and concepts as follows:

[2] These conditions are enforced to attain decidability. We usually call an RBox to be *regular* because of the first condition.

$$
\begin{array}{rl}
(R^-)^\mathcal{I} = & \{(x,y) \mid (y,x) \in R^\mathcal{I}\} \\
(\neg C)^\mathcal{I} = & \Delta \setminus C^\mathcal{I} \\
(\exists S.\mathsf{Self})^\mathcal{I} = & \{x \mid (x,x) \in S^\mathcal{I}\} \\
(C \sqcap D)^\mathcal{I} = & C^\mathcal{I} \cap D^\mathcal{I} \\
(C \sqcup D)^\mathcal{I} = & C^\mathcal{I} \cup D^\mathcal{I} \\
(\forall R.C)^\mathcal{I} = & \{p_1 \in \Delta \mid \forall p_2.(p_1,p_2) \in R^\mathcal{I} \rightarrow p_2 \in C^\mathcal{I}\} \\
(\exists R.C)^\mathcal{I} = & \{p_1 \in \Delta \mid \exists p_2.(p_1,p_2) \in R^\mathcal{I} \wedge p_2 \in C^\mathcal{I}\} \\
(\leq nS.C)^\mathcal{I} = & \{p_1 \in \Delta \mid \#\{p_2 \mid (p_1,p_2) \in S^\mathcal{I} \wedge p_2 \in C^\mathcal{I}\} \leq n\} \\
(\geq nS.C)^\mathcal{I} = & \{p_1 \in \Delta \mid \#\{p_2 \mid (p_1,p_2) \in S^\mathcal{I} \wedge p_2 \in C^\mathcal{I}\} \geq n\} \\
\{a_1,\ldots,a_n\}^\mathcal{I} = & \{a_1^\mathcal{I},\ldots,a_n^\mathcal{I}\}
\end{array} \quad \diamondsuit
$$

where C, D are $\mathcal{SROIQ}$-concepts, R, S are roles, n is a non-negative integer and $\#M$ represents the cardinality of the set M.

Given an axiom α (TBox, RBox or ABox axiom), we say the an interpretation $\mathcal{I}$ satisfies α, written $\mathcal{I} \models \alpha$, if it satisfies the condition given in Table 1. Similarly $\mathcal{I}$ satisfies a TBox $\mathcal{T}$, written $\mathcal{I} \models \mathcal{T}$, if it satisfies all the axioms in $\mathcal{T}$. The satisfaction of an RBox and an ABox by an interpretation is defined in the same way. We say $\mathcal{I}$ satisfies a knowledge base $\Sigma = (\mathcal{T}, \mathcal{R}, \mathcal{A})$ if it satisfies $\mathcal{T}$, $\mathcal{R}$ and $\mathcal{A}$. We write $\mathcal{I} \models \Sigma$. We call $\mathcal{I}$ a model of Ψ. A knowledge base is said to be *consistent* if it has a model.

We now present the extension of the DL $\mathcal{SROIQ}$ by the epistemic operator $\mathbf{K}$. We call this extension $\mathcal{SROIQK}$.

2.2 K-extensions of $\mathcal{SROIQ}$

The embedding of the epistemic operator $\mathbf{K}$ into the description logic $\mathcal{ALC}$ was first proposed in [**?**]. The logic obtained is called $\mathcal{ALCK}$. A similar approach has been taken in [**?**], which we follow in this work. We consider $\mathcal{SROIQ}$ as the basis DL and call its $\mathbf{K}$-extension $\mathcal{SROIQK}$. In $\mathcal{SROIQK}$ we allow $\mathbf{K}$ in front of the concepts and role names. In the following we provide the formal syntax and semantics of such language where N_C, N_R, N_I, $\mathbf{R}$ are as in Definition 1.

Definition 3. A $\mathcal{SROIQK}$-role is defined as follows:

– every $R \in \mathbf{R}$ is a $\mathcal{SROIQK}$-role;

Table 1. Semantics of $\mathcal{SROIQ}$ axioms

Axiom α	$\mathcal{I} \models \alpha$, if
$R_1 \circ \cdots \circ R_n \sqsubseteq R$	$R_1^{\mathcal{I}} \circ \cdots \circ R_n^{\mathcal{I}} \subseteq R^{\mathcal{I}}$
$\mathsf{Tra}(\mathrm{R})$	$R^{\mathcal{I}} \circ R^{\mathcal{I}} \subseteq R^{\mathcal{I}}$
$\mathsf{Ref}(\mathrm{R})$	$(x, x) \in R^{\mathcal{I}}$ for all $x \in \Delta^{\mathcal{I}}$
$\mathsf{Irr}(\mathrm{S})$	$(x, x) \notin S^{\mathcal{I}}$ for all $x \in \Delta^{\mathcal{I}}$
$\mathsf{Dis}(\mathrm{S,T})$	$(x, y) \in S^{\mathcal{I}}$ implies $(x, y) \notin T^{\mathcal{I}}$ for all $x, y \in \Delta^{\mathcal{I}}$
$\mathsf{Sym}(\mathrm{S})$	$(x, y) \in S^{\mathcal{I}}$ implies $(y, x) \in S^{\mathcal{I}}$ for all $x, y \in \Delta^{\mathcal{I}}$
$\mathsf{Asy}(\mathrm{S})$	$(x, y) \in S^{\mathcal{I}}$ implies $(x, y) \notin S^{\mathcal{I}}$ for all $x, y \in \Delta^{\mathcal{I}}$
$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$
$R(a, b)$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$
$a \doteq b$	$a^{\mathcal{I}} = a^{\mathcal{I}}$
$a \not\doteq b$	$a^{\mathcal{I}} \neq b^{\mathcal{I}}$

– if R is a $\mathcal{SROIQK}$-role than so are $\mathsf{K}R$ and R^-.

We call a $\mathcal{SROIQK}$-role an *epistemic role* if **K** occurs in it. An epistemic role is *simple* if it is of the form $\mathbf{K}S$ where S is a simple $\mathcal{SROIQ}$-role. Now $\mathcal{SROIQK}$-concepts are defined as follows:

– every $\mathcal{SROIQ}$-concept is an $\mathcal{SROIQ}$-concept;
– if C and D are $\mathcal{SROIQK}$-concepts, and S and R are $\mathcal{SROIQK}$ roles with S being simple, then the following are $\mathcal{SROIQK}$-concepts:

$$\mathsf{K}C \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid \forall R.C \mid \exists R.C \mid \leq nS.C \mid \geq nS.C \qquad \diamondsuit$$

The semantics of $\mathcal{SROIQK}$ is given as *possible world semantics* in terms of *epistemic interpretations*. Thereby following assumptions are made:

1. all interpretations are defined over a fixed infinite domain Δ (Common Domain Assumption);
2. for all interpretations, the mapping from individuals to domains elements is fixed: it is just the identity function (Rigid Term Assumption).

Definition 4. An epistemic interpretation for $\mathcal{SROIQK}$ is a pair $(\mathcal{I}, \mathcal{W})$ where $\mathcal{I}$ is a $\mathcal{SROIQ}$-interpretation and $\mathcal{W}$ is a set of $\mathcal{SROIQ}$-interpretations, where $\mathcal{I}$ and all of $\mathcal{W}$ have the same infinite domain Δ with $N_I \subset \Delta$. The interpretation function $\cdot^{\mathcal{I},\mathcal{W}}$ is then defined as follows:

$$
\begin{array}{rcl}
a^{\mathcal{I},\mathcal{W}} = & a & \text{for } a \in N_I \\
A^{\mathcal{I},\mathcal{W}} = & A^{\mathcal{I}} & \text{for } A \in N_C \\
R^{\mathcal{I},\mathcal{W}} = & R^{\mathcal{I}} & \text{for } R \in N_R \\
\top^{\mathcal{I},\mathcal{W}} = & \Delta & \text{(the domain of } \mathcal{I}) \\
\bot^{\mathcal{I},\mathcal{W}} = & \emptyset & \\
(C \sqcap D)^{\mathcal{I},\mathcal{W}} = & C^{\mathcal{I},\mathcal{W}} \cap D^{\mathcal{I},\mathcal{W}} & \\
(C \sqcup D)^{\mathcal{I},\mathcal{W}} = & C^{\mathcal{I},\mathcal{W}} \cup D^{\mathcal{I},\mathcal{W}} & \\
(\neg C)^{\mathcal{I},\mathcal{W}} = & \Delta \setminus C^{\mathcal{I},\mathcal{W}} & \\
(\forall R.C)^{\mathcal{I},\mathcal{W}} = & \{p_1 \in \Delta \mid \forall p_2.(p_1, p_2) \in R^{\mathcal{I},\mathcal{W}} \to p_2 \in C^{\mathcal{I},\mathcal{W}}\} & \\
(\exists R.C)^{\mathcal{I},\mathcal{W}} = & \{p_1 \in \Delta \mid \exists p_2.(p_1, p_2) \in R^{\mathcal{I},\mathcal{W}} \land p_2 \in C^{\mathcal{I},\mathcal{W}}\} & \\
(\leq nR.C)^{\mathcal{I},\mathcal{W}} = & \{d \mid \#\{e \in C^{\mathcal{I},\mathcal{W}} \mid (d, e) \in R^{\mathcal{I},\mathcal{W}}\} \leq n\} & \\
(\geq nR.C)^{\mathcal{I},\mathcal{W}} = & \{d \mid \#\{e \in C^{\mathcal{I},\mathcal{W}} \mid (d, e) \in R^{\mathcal{I},\mathcal{W}}\} \geq n\} & \\
(\mathbf{K}C)^{\mathcal{I},\mathcal{W}} = & \bigcap_{\mathcal{J} \in \mathcal{W}} (C^{\mathcal{J},\mathcal{W}}) & \\
(\mathbf{K}R)^{\mathcal{I},\mathcal{W}} = & \bigcap_{\mathcal{J} \in \mathcal{W}} (R^{\mathcal{J},\mathcal{W}}) &
\end{array}
$$

where C and D are $\mathcal{SROIQK}$-concepts and R is a $\mathcal{SROIQK}$-role. Further for an epistemic role $(\mathbf{K}R)^-$, we set $[(\mathbf{K}R)^-]^{\mathcal{I}} := (\mathbf{K}R^-)^{\mathcal{I}}$. ◊

From the above one can see that $\mathbf{K}C$ is interpreted as the set of objects that are in the interpretation of C under every interpretation in $\mathcal{W}$. Note that the rigid term assumption implies the unique name assumption (UNA) i.e., for any interpretation $\mathcal{I} \in \mathcal{W}$ and for any two distinct individual names a and b we have that $a^{\mathcal{I}} \neq b^{\mathcal{I}}$.

The notions of GCI, assertion, role hierarchy, ABox, TBox and knowledge base, and their interpretations as defined in Definition 1 and 2 can be extended to that of $\mathcal{SROIQK}$ by allowing for $\mathcal{SROIQK}$-concepts and $\mathcal{SROIQK}$-roles in their definitions.

An *epistemic model* for a $\mathcal{SROIQK}$-knowledge base $\Psi = (\mathcal{T}, \mathcal{R}, \mathcal{A})$ is a *maximal* non-empty set $\mathcal{W}$ of $\mathcal{SROIQ}$-interpretations such that $(\mathcal{I}, \mathcal{W})$ satisfies $\mathcal{T}$, $\mathcal{R}$ and $\mathcal{A}$ for each $\mathcal{I} \in \mathcal{W}$. A $\mathcal{SROIQK}$-knowledge base Ψ is said to be *satisfiable* if it has an epistemic model. The knowledge base Ψ *entails* an axiom φ, written $\Psi \models \varphi$, if for every epistemic model $\mathcal{W}$ of Ψ, we have that for every $\mathcal{I} \in \mathcal{W}$, the epistemic interpretation $(\mathcal{I}, \mathcal{W})$ satisfies φ. By definition every $\mathcal{SROIQ}$-knowledge base is an $\mathcal{SROIQK}$-knowledge base. Note that a given $\mathcal{SROIQ}$-knowledge base Σ has up to isomorphism only one unique epistemic model which is the set of all models of Σ having infinite domain and satisfying the unique name assumption. We denote this model by $\mathcal{M}(\Sigma)$.

3 Answering Epistemic Queries

In this section we introduce the notion of answer to an epistemic query posed to a knowledge base. For reasons that we will discuss in Section 4, we restrict the underlying knowledge base to $\mathcal{SRIQ}$, i.e., we disallow the use of nominals, the universal role as well as the epistemic operator K. On the other hand we allow for a richer language, namely $\mathcal{SROIQK}$ for querying this knowledge base.

Definition 5. (Epistemic Queries)

An *epistemic query* is of the form $C(a)$? where C is a $\mathcal{SROIQK}$-concept and a is an individual. Given a knowledge base Σ, the answer to the query $C(a)$? posed to Σ is:

1. YES, if $\Sigma \models\!\!\!\models C(a)$
2. NO, if $\Sigma \models\!\!\!\models \neg C(a)$
3. $\mathsf{UNKNOWN}$, otherwise.

Let N_I be the set of individuals occurring in Σ, then the *answer set* of C w.r.t. Σ is the set of individuals $\{a \in N_I \mid \Sigma \models\!\!\!\models C(a)\}$. ♢

In the subsequent paragraphs, we show that such an epistemic query posed to an $\mathcal{SRIQ}$-knowledge base can answered using standard reasoning in the DL $\mathcal{SROIQ}$.

In this work we do not discuss the question of the importance of considering such a query language. For such discussion, we refer the readers to [**?**,**?**]. We rather focus on our objective, i.e., to use standard reasoners to answer epistemic queries. In the following we provide an example (originally taken from [**?**]) to demonstrate our method of answering epistemic queries.

Example 6. Consider the knowledge base Σ which is presented pictorially in Figure 6. For the sake of simplicity, we represent the interpretation of the individual name mary by Mary, of bob by Bob, of susan by Susan and of peter by Peter. Similarly for the individual names cs221, cs324 and ee282 we represent their interpretations by CS221, CS324 and EE282 respectively. Now we proceed by considering different queries.

Query 1 $\Sigma \models\!\!\!\models \exists \mathsf{K}\mathsf{ENROLLED}.\mathsf{K}\mathsf{Grad}(ee282)$?

Query 1 asks if Σ knows about an individual who is known to be a graduate student as well as known to be enrolled in the course EE282.

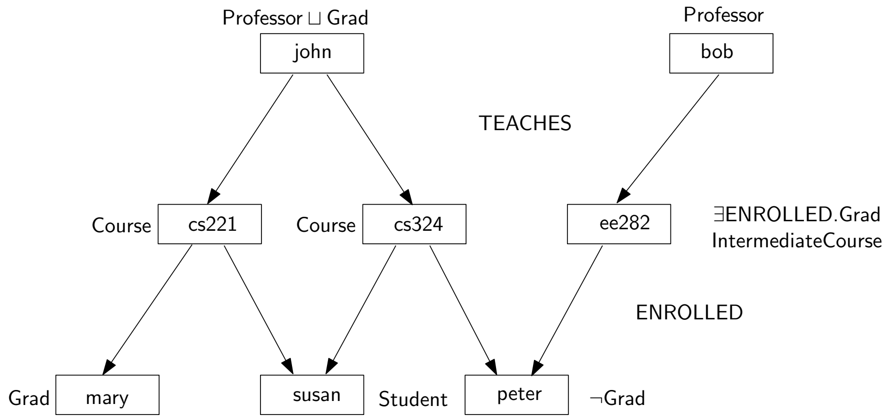

Fig. 1. Pictorial Representation of Σ

The answer is NO and one can verify this by using the tableau techniques[3] presented in [**?**]. But using the method we present in this work, we can reduce this query to a non-epistemic query. First of all we compute the set of all individuals that are known to be graduate, i.e., the interpretation of the concept **K**Grad. The only known graduate is Mary as it is explicitly asserted by Σ. Peter is explicitly asserted to be non-graduate whereas Susan is asserted to be a student and no further information is provided about her. In other words, there can models where Susan is a graduate and other models where she is a non-graduate. Hence she does not belong to the interpretation of the concept Grad in all possible worlds (interpretations) i.e., she does not belong to the interpretation of **K**Grad. What we do next is to compute all courses in which Mary is enrolled in every possible worlds i.e., we compute the set

$$\{a \in N_I \mid \Sigma \Vdash \mathsf{ENROLLED}(a, \mathsf{mary})\}$$

According to Figure 6 this yields {cs221}. Hence Query 1 can be answered by answering the non-epistemic query {cs221}(ee282)? which of course is NO. Note that in answering Query 1, instead of dealing with the operator **K** in a way proposed in different literature, we simply do some instance retrievals and entailment checking. For example, the set of all individuals known to be graduate can be computed by retrieving all the instances of the concept Grad.

Query 2 $\Sigma \Vdash \forall \mathbf{K}\mathsf{TEACHES}.\mathbf{K}\mathsf{IntermedidateCourse}(bob)$? ◊

[3] In the cited paper, an tableau algorithm for $\mathcal{ALCK}$ has been presented.

Query 2 asks if everything known to be taught by Bob is also known to be an intermediate course. From Σ it follows that this is the case i.e., the answer to the query is YES. Similar to answering Query 1, we retrieve the instance of the concept $\mathsf{KIntermediateCourse}$, which in this case is only $\mathsf{ee282}$. Similarly we can compute all people who are known to teach EE282 (the only known intermediate course). In this case Bob is the only person with this property. Note that there can be models in which John also teaches an intermediate course. But we are interested in people who are known to teach only known intermediate courses and Bob is the only such candidate in this case. Now Query 2 can be rephrased as $\{bob\}(bob)$? Since $\Sigma \models \{\mathsf{bob}\}(\mathsf{bob})$ therefore, the answer is YES.

In Example 6, we have seen how one can reduce an epistemic query to standard DL query. However, one has to be careful that a concept, for which we are retrieving all the instances, should not be equivalent to the top concept i.e., $C \not\equiv \top$. If that is the case, instead of replacing an epistemic concept within an epistemic query with a set of individual, we simply replace it with $\top$ and proceed in the usual way. A similar check one has to make when dealing with a role of the form $\mathbf{K}R$ where R is universal. The reason is simply that in the following, we prove Lemma 15 and Lemma 18 where we mainly depend on elements of the domain which are not the interpretation of some individual occurring in the KB. To prove the correctness of our method, we introduce a translation function, that takes a $\mathcal{SROIQK}$-concept and returns an $\mathcal{SROIQ}$-concept. Note that this translation is not syntactic in the sense that, we have to make intermediate calls to a reasoner. Given a $\mathcal{SRIQ}$-KB Σ, we denote the set of all individual occurring in Σ by N_I. By an individual a to occur in Σ we mean that a occurs in an assertion or in a concept in Σ. Based on N_I, we categorize the elements of the domain of an interpretation into two types. An element x is called *named* w.r.t. to an interpretation $\mathcal{I}$ if there is an individual $a \in N_I$ such that $a^{\mathcal{I}} = x$, else it is called *anonymous*. Since a $\mathcal{SRIQ}$-knowledge base has a unique epistemic model, we just say named or anonymous elements instead of referring to a particular interpretation. In order to take care of the unique name assumption that is hard-wired in the epistemic semantics we have to axiomatize this assumption in the underlying knowledge base as described in the following.

Definition 7. Given a $\mathcal{SRIQ}$ knowledge base Σ, we denote by Σ_{UNA} the knowledge base $\Sigma \cup \{a \not\approx b \mid a, b \in N_I, a \neq b\}$. $\Diamond$

Note that due to the additional axioms Σ_{UNA}, the set of models of Σ is a superset of that of Σ_{UNA}.

Fact 8. *The set of models of Σ_{UNA} is exactly the set of those models of Σ that satisfy the unique name assumption.*

Now we present the translation function, which takes a $\mathcal{SROIQK}$-concept as an input and translates it into a $\mathcal{SROIQ}$-concept. It is this function that we use in reducing epistemic queries to the non-epistemic ones. Later on, in Lemma 15 and 18, we prove that the extension of a concept is preserved before and after the translation. Also note that in the definition of this function, we require a number of entailment checking and instance retrieval. Hence it is not a pure syntactic translation function.

Definition 9. (Translation Function Φ)
Given a $\mathcal{SRIQ}$ knowledge base Σ and a $\mathcal{SROIQK}$-concept C, the translation of C, denoted by $\Phi(C)$, with respect to Σ is given as follows:

$$
\Phi(C) = \begin{cases}
C & \text{if } C \text{ is an atomic or one-of concept, } \exists S.\mathsf{Self}, \top \text{ or } \bot; \\
\top & \text{if } C = \mathbf{K}D \text{ and } \Sigma_{\text{UNA}} \models D \equiv \top \\
\{a \in N_I \mid \Sigma_{\text{UNA}} \models S(a,a)\} & \text{if } C = \exists \mathbf{K} S.\mathsf{Self} \\
\{a \in N_I \mid \Sigma_{\text{UNA}} \models \Phi(D)(a)\} & \text{if } C = \mathbf{K}D \text{ and } \Sigma_{\text{UNA}} \not\models D \equiv \top; \\
\Phi(C_1) \sqcap \Phi(C_2) & \text{if } C = C_1 \sqcap C_2; \\
\Phi(C_1) \sqcup \Phi(C_2) & \text{if } C = C_1 \sqcup C_2; \\
\neg \Phi(C_1) & \text{if } C = \neg C_1; \\
\exists R.\Phi(D) & \text{if } C = \exists R.D \text{ for non-epistemic role } R; \\
\{a \in N_I \mid \exists b \in N_I \text{ such that } \Sigma_{\text{UNA}} \models P(a,b) \wedge \Sigma_{\text{UNA}} \models \Phi(D)(b)\} & \text{if } C = \exists \mathbf{K} P.D; \\
\forall R.\Phi(D) & \text{if } C = \forall R.D \text{ for non-epistemic role } R; \\
\{a \in N_I \mid \forall b \in N_I \text{ such that } \Sigma_{\text{UNA}} \models P(a,b) \rightarrow \Sigma_{\text{UNA}} \models \Phi(D)(b)\} & \text{if } C = \forall \mathbf{K} P.D; \\
\geq nS.\Phi(D) & \text{if } C = \geq nS.D \text{ for non-epistemic role } S; \\
\{a \in N_I \mid \#\{b \in N_I \text{ such that } \Sigma_{\text{UNA}} \models \Phi(D)(b) \wedge \Sigma_{\text{UNA}} \models S(a,b)\} \geq n\} & \text{if } C = \geq n\mathbf{K} S.D; \\
\leq nS.\Phi(D) & \text{if } C = \leq nS.D \text{ for non-epistemic role } S; \\
\{a \in N_I \mid \#\{b \in N_I \text{ such that } \Sigma_{\text{UNA}} \models \Phi(D)(b) \wedge \Sigma_{\text{UNA}} \models S(a,b)\} \leq n\} & \text{if } C = \leq n\mathbf{K} S.D \\
\Xi U.\Phi(D) & \text{if } C = \Xi \mathbf{K} U.D \text{ with } \Xi \in \{\forall, \exists, \geq n, \leq n\}
\end{cases}
\quad \Diamond
$$

where D is an epistemic concept, R and P are roles, S is a simple role and n is a non-negative interger. This function is exactly the reduction from an epistemic query to a non-epistemic query. In fact, it takes as an input a $\mathcal{SROIQK}$-concept and returns a $\mathcal{SROIQ}$-concept as an output. Consider Query 1 given in Example 6. We apply the translation function to the concept in this query i.e. $\Phi(\exists \mathbf{K}\mathsf{ENROLLED}.\mathbf{K}\mathsf{Grad})$, and hence get

$$\{a \in N_I \mid \exists b \in N_I.\Sigma_{\text{UNA}} \models \mathsf{ENROLLED}(a,b) \rightarrow \Sigma_{\text{UNA}} \models \Phi(\mathbf{K}\mathsf{Grad})(b)\}$$

As $\Phi(\mathbf{K}\mathsf{Grad}) = \{a \in N_I \mid \Sigma_{\text{UNA}} \models \mathsf{Grad}(a)\} = \{\mathsf{mary}\}$, therefore we get

$$\{a \in N_I \mid \exists b \in N_I.\Sigma_{\text{UNA}} \models \mathsf{ENROLLED}(a,b) \rightarrow \Sigma_{\text{UNA}} \models \{\mathsf{mary}\}(b)\}$$

Hence the resultant concept is $\{\mathsf{cs221}\}$ and the original query can be answered by answering $\{\mathsf{cs221}\}(\mathsf{ee282})$? which, of course, is NO because the only course in which Mary is enrolled in, is cs221 and the unique name assumption holds. Again one thing needs to be noted that this is not a syntactic translation purely. It involves calls to a reasoner in order to retrieve instances of some concepts or check some entailments.

In the following subsections, we prove the correctness of this method. Note that our method can be thought of as a reduction from epistemic queries to non-epistemic queries through some computation, i.e., we need to do some reasoning on our knowledge base (in this case checking some entailments and instance retrieval) in order to answer the original query. The important point is that our translation depends on the set of individuals occurring in the knowledge base. For an epistemic concept C which is not equivalent to $\top$, one can deal with $\mathbf{K}$ by considering individuals occurring in the knowledge base only (see Definition 9). To prove this formally, we introduce some definitions.

Definition 10. Given an interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, a set Δ with $N_I \subseteq \Delta$, and a bijection $\varphi : \Delta^{\mathcal{I}} \rightarrow \Delta$ with $\varphi(a^{\mathcal{I}}) = a$ for all $a \in N_I$, the *renaming* of $\mathcal{I}$ according to φ, denoted by $\varphi(\mathcal{I})$, is defined as the interpretation $(\Delta, \cdot^{\varphi(\mathcal{I})})$ with:

- $a^{\varphi(\mathcal{I})} = \varphi(a^{\mathcal{I}}) = a$ for every individual name a
- $A^{\varphi(\mathcal{I})} = \{\varphi(z) \mid z \in A^{\mathcal{I}}\}$ for every concept name A
- $P^{\varphi(\mathcal{I})} = \{(\varphi(z), \varphi(w)) \mid (z,w) \in P^{\mathcal{I}}\}$ for every role name P ◊

Now we prove in the following that...

Lemma 11. *Let Σ be a $\mathcal{SRIQ}$ knowledge base and let $\mathcal{I}$ be a model of Σ with infinite domain. Then, every renaming $\varphi(\mathcal{I})$ of $\mathcal{I}$ satisfies $\varphi(\mathcal{I}) \in \mathcal{M}(\Sigma)$.*

Proof. By definition, the renaming satisfies the common domain and rigid term assumption. Modelhood w.r.t. Σ immediately follows from the isomorphism lemma of first-order interpretations [?] since $\mathcal{I}$ and $\varphi(\mathcal{I})$ are isomorphic and φ is an isomorphism from $\mathcal{I}$ to $\varphi(\mathcal{I})$. □

It is this property that we consider in the definition of Φ; we consider set of named elements by looking for individual in N_I (see Definition 9). In Lemma 11 we show that whenever an element x belongs to the interpretation of an epistemic concept of the form $\mathbf{K}C$ with $C \not\equiv \top$ then the element needs to be named and the knowledge base entails the assertion $C(a)$ where a is the individual name interpreted by x. A similar property is shown for the role names in Lemma 18. The proof in each case depends on the assumption that the domain of interpretations under consideration is infinite. In order to allow for such assumptions we prove in the following lemma that any finite model[4] of a $\mathcal{SRIQ}$ knowledge base can be lifted to an infinite model of it. Before stating the lemma we need the following notion.

Definition 12. For any finite $\mathcal{SRIQ}$ interpretation $\mathcal{I}$, the *lifting* of $\mathcal{I}$ to ω is the interpretation $\mathcal{I}_\omega$ defined as follows:

- $\Delta^{\mathcal{I}_\omega} := \Delta^{\mathcal{I}} \times \mathbb{N}$,
- $a^{\mathcal{I}_\omega} := \langle a^{\mathcal{I}}, 0\rangle$ for every $a \in N_I$,
- $A^{\mathcal{I}_\omega} := \{\langle x, i\rangle \mid x \in A^{\mathcal{I}} \text{ and } i \in \mathbb{N}\}$ for each concept name $A \in N_C$,
- $R^{\mathcal{I}_\omega} := \{(\langle x, i\rangle, \langle x', i\rangle) \mid (x, x') \in R^{\mathcal{I}} \text{ and } i \in \mathbb{N}\}$ for every role name $R \in N_R$. $\diamondsuit$

Lemma 13. *For all $\langle x, i\rangle \in \Delta^{\mathcal{I}_\omega}$ and all $\mathcal{SRIQ}$-concepts C that $\langle x, i\rangle \in C^{\mathcal{I}_\omega}$ if and only if $x \in C^{\mathcal{I}}$.*

Proof. The proof is by the induction on the structure of C:

- For the atomic concept, $\top$ or $\bot$ it follows immediately from the definition of $\mathcal{I}_\omega$.
- Let $C = \neg D$. For any $x \in \Delta^{\mathcal{I}}$ we have that
  $x \in (\neg D)^{\mathcal{I}}$
  $\Leftrightarrow x \notin D^{\mathcal{I}}$
  $\Leftrightarrow \langle x, i\rangle \notin D^{\mathcal{I}_\omega}$ for $i \in \mathbb{N}$ (Induction)
  $\Leftrightarrow \langle x, i\rangle \in (\neg D)^{\mathcal{I}_\omega}$ for $i \in \mathbb{N}$.
- Let $C = C_1 \sqcap C_2$. For any $x \in \Delta^{\mathcal{I}}$ we have that
  $x \in (C_1 \sqcap C_2)^{\mathcal{I}}$
  $\Leftrightarrow x \in C_1^{\mathcal{I}}$ and $x \in C_2^{\mathcal{I}}$
  $\Leftrightarrow \langle x, i\rangle \in C_1^{\mathcal{I}_\omega}$ and $\langle x, i\rangle \in C_2^{\mathcal{I}_\omega}$ for $i \in \mathbb{N}$ (Induction)
  $\Leftrightarrow \langle x, i\rangle \in (C_1 \sqcap C_2)^{\mathcal{I}_\omega}$ for $i \in \mathbb{N}$.

[4] by a finite model we mean a model with finite domain

– Let $C = \exists R.D$ for $R \in \mathbf{R}$. For any $x \in \Delta^{\mathcal{I}}$ we have that
$x \in (\exists R.D)^{\mathcal{I}}$
$\Leftrightarrow$ there is a $y \in \Delta^{\mathcal{I}}$ such that $(x, y) \in R^{\mathcal{I}}$ and $y \in D^{\mathcal{I}}$
$\Leftrightarrow$ there is $\langle y, i\rangle \in \Delta^{\mathcal{I}_\omega}$ for $i \in \mathbb{N}$ with $(\langle x, i\rangle, \langle y, i\rangle) \in R^{\mathcal{I}_\omega}$ and $\langle y, i\rangle \in D^{\mathcal{I}_\omega}$ (Def 12 and Induction)
$\Leftrightarrow \langle x, i\rangle \in (\exists R.D)^{\mathcal{I}_\omega}$
– The rest of the cases can be proved analogously.

Lemma 14. *Let Σ be a $\mathcal{SRIQ}$ knowledge base. For any interpretation $\mathcal{I}$ we have that*

$$\mathcal{I} \models \Sigma \text{ if and only if } \mathcal{I}_\omega \models \Sigma$$

Proof. First we note that it follows immediately from the definition of $\mathcal{I}_\omega$ that for any $\mathcal{SRIQ}$-role $R \in \mathbf{R}$ and $(\langle x, i\rangle, \langle y, i'\rangle) \in \Delta^{\mathcal{I}_\omega}$ for $i, i' \in \mathbb{N}$ we have that $(\langle x, i\rangle, \langle y, i'\rangle) \in R^{\mathcal{I}_\omega}$ if an only if $(x, y) \in R^{\mathcal{I}}$ and $i = i'$ for an interpretation $\mathcal{I}$. Now for any RIA $R_1 \circ \ldots R_n \sqsubseteq R$ we have that:
$\mathcal{I} \models R_1 \circ \ldots R_n \sqsubseteq R$
$\Leftrightarrow \mathcal{I} \models R_1^{\mathcal{I}} \circ \ldots R_n^{\mathcal{I}} \subseteq R^{\mathcal{I}}$
$\Leftrightarrow$ for any $x_0, \ldots, x_n \in \Delta^{\mathcal{I}}$, whenever $(x_{i-1}, x_i) \in R_i^{\mathcal{I}}$ for $1 \leq i \leq n$ then $(x_0, x_n) \in R^{\mathcal{I}}$
$\Leftrightarrow$ for any $x_0, \ldots, x_n \in \Delta^{\mathcal{I}}$ and any $j \in \mathbb{N}$, whenever $(\langle x_{i-1}, j\rangle, \langle x_i, j\rangle) \in R_i^{\mathcal{I}_\omega}$ for $1 \leq i \leq n$ then $(\langle x_0, j\rangle, \langle x_n, j\rangle) \in R^{\mathcal{I}_\omega}$
$\Leftrightarrow \mathcal{I}_\omega \models R_1 \circ \ldots R_n \sqsubseteq R$.
The second last equivalence holds as $(x_{i-1}, x_i) \in R_i^{\mathcal{I}}$ for $1 \leq i \leq n$ and any non-negative integer j implies that $(\langle x_{i-1}, j\rangle, \langle x_i, j\rangle) \in R_i^{\mathcal{I}_\omega}$. Similary $(\langle x_{i-1}, j_{i-1}\rangle, \langle x_i, j_i\rangle) \in R^{\mathcal{I}_\omega}$ for $1 \leq i \leq n$ implies that $(x_{i-1}, x_i) \in R^{\mathcal{I}}$ and that all j_i, s are equal. And the same holds for the role R.

Similary, for any role characteristic $\mathsf{Ref}(R)$, we have that:
$\mathcal{I} \models \mathsf{Ref}(R)$
$\Leftrightarrow (x, x) \in R^{\mathcal{I}}$ for all $x \in \Delta^{\mathcal{I}}$
$\Leftrightarrow (\langle x, j\rangle, \langle x, j\rangle) \in R^{\mathcal{I}_\omega}$ for any $j \in \mathbb{N}$ and $x \in \Delta^{\mathcal{I}}$
$\Leftrightarrow (\langle x, j\rangle, \langle x, j\rangle) \in R^{\mathcal{I}_\omega}$ for any $\langle x, j\rangle \in \Delta^{\mathcal{I}_\omega}$ as $\Delta^{\mathcal{I}_\omega} = \Delta^{\mathcal{I}} \times \mathbb{N}$
$\Leftrightarrow \mathcal{I}_\omega \models \mathsf{Ref}(R)$.
In the same way, we can prove for any of the rest of the role characteristics that whenever $\mathcal{I}$ models it so does $\mathcal{I}_\omega$. Consequently we have that for any role hierarchy $\mathcal{R}$, $\mathcal{I} \models \mathcal{R}$ if and only if $\mathcal{I}_\omega \models \mathcal{R}$.

Invoking Lemma 13, we get that for any GCI $C \sqsubseteq D$ and for any interpretation $\mathcal{I}$, $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ if and only if $C^{\mathcal{I}_\omega} \subseteq D^{\mathcal{I}_\omega}$. Further for any TBox $\mathcal{T}$, $\mathcal{I} \models \mathcal{T}$ if and only if $\mathcal{I}_\omega \models \mathcal{T}$.

Finally for an ABox $\mathcal{A}$ we show that for each assertion in $\alpha \in \mathcal{A}$, $\mathcal{I} \models \alpha$ if and only if $\mathcal{I}_\omega \models \alpha$.

- α is of the form $C(a)$: Now for an interpretation $\mathcal{I}$ it follows from the definition of $\mathcal{I}_\omega$ that $a^{\mathcal{I}_\omega} = (a^{\mathcal{I}}, 0)$. As we have already shown that $a^{\mathcal{I}} \in C^{\mathcal{I}}$ if and only if $(a^{\mathcal{I}}, i) \in C^{\mathcal{I}_\omega}$ for $i \in \mathbb{N}$. Hence we get that $a^{\mathcal{I}} \in C^{\mathcal{I}}$ if and only if $(a^{\mathcal{I}}, 0) \in C^{\mathcal{I}_\omega}$.
- Analogously we can show an interpretation $\mathcal{I}$ satisfies an assertion if and only if $\mathcal{I}_\omega$ does so.

In the following, note that the property we prove in Lemma 15 is just for the concepts not equivalent to $\top$.

Lemma 15. *Let Σ be a $\mathcal{SRIQ}$ knowledge base. For any epistemic concept $C =$***K***D with $\Sigma_{\mathrm{UNA}} \not\models D \equiv \top$ and $x \in \Delta$, we have that $x \in C^{\mathcal{I},\mathcal{M}(\Sigma)}$ iff x is named such that there is an individual $a \in N_I$ with $x = a^{\mathcal{I},\mathcal{M}(\Sigma)}$ and $\Sigma_{\mathrm{UNA}} \models D(a)$.*

Proof. " $\Rightarrow$ "
Suppose that $x \in C^{\mathcal{I},\mathcal{M}(\Sigma)}$. It means that

$$x \in \bigcap_{\mathcal{J} \in \mathcal{M}(\Sigma)} D^{\mathcal{J}}$$

To the contrary, suppose that there is no $a \in N_I$ such that $a^{\mathcal{I},\mathcal{M}(\Sigma)} = x$ and $\Sigma_{\mathrm{UNA}} \models D(a)$ i.e., x is an anonymous element. Since $\Sigma_{\mathrm{UNA}} \not\models \top \equiv D$, there is a model $\mathcal{I}'$ of Σ_{UNA} such that $D^{\mathcal{I}'} \neq \Delta^{\mathcal{I}'}$. This implies that there is a $y \in \Delta^{\mathcal{I}'}$ such that $y \notin D^{\mathcal{I}'}$. Considering $\mathcal{I}'_\omega$, we can invoke Lemma 14 to ensure $\mathcal{I}'_\omega \models \Sigma_{\mathrm{UNA}}$, moreover Lemma 13 guarantees $\langle y, 1\rangle \notin D^{\mathcal{I}'_\omega}$. On the other hand, by construction, $\langle y, 1\rangle$ is anonymous. Let $\varphi : \Delta^{\mathcal{I}'} \times \mathbb{N} \to \Delta$ be a bijection such that $\varphi(a^{\mathcal{I}'_\omega}) = a^{\mathcal{I}}$ for all $a \in N_I$ and $\varphi(\langle y, 1\rangle) = x$. Such a φ exists, as $|\Delta^{\mathcal{I}'} \times \mathbb{N}| = |\Delta|$ and $\mathcal{I}'_\omega$ satisfies the unique name assumption. By Lemma 11, we get that $\varphi(\mathcal{I}'_\omega) \in \mathcal{M}(\Sigma)$. By the choice of φ we get $x \notin D^{\varphi(\mathcal{I}'_\omega)}$ due to $\langle y, 1\rangle \notin D^{\mathcal{I}'_\omega}$ and the fact that φ is an isomorphism. In particular,

$$x \notin \bigcap_{\mathcal{J} \in \mathcal{M}(\Sigma)} D^{\mathcal{J}}$$

which is a contradiction.
" $\Leftarrow$ "
Suppose there is $a \in N_I$ such that $a^{\mathcal{I},\mathcal{M}(\Sigma)} = x$ and $\Sigma_{\mathrm{UNA}} \models D(a)$. This implies that for any $\mathcal{I} \in \mathcal{M}(\Sigma)$ we have that $x \in D^{\mathcal{I}}$. Hence we get that $x \in \mathbf{K}D^{\mathcal{I},\mathcal{M}(\Sigma)}$.

A similar property can be proved for the roles as well. But before that we make the following claim.

Claim 16. *Let Σ be a knowledge base. For any universal role R we have:*

$$\mathbf{K}R^{\mathcal{I},\mathcal{M}(\Sigma)} = R^{\mathcal{I},\mathcal{M}(\Sigma)}$$

The claim follows trivially as R is universal and therefore $R^{\mathcal{J}} = \Delta \times \Delta$ for any $\mathcal{J} \in \mathcal{M}(\Sigma)$. It means that $\bigcap_{\mathcal{J}\in\mathcal{M}(\Sigma)} R^{\mathcal{J}} = \Delta \times \Delta$.

As in the case of concepts, whenever an epistemic concept contains a role of the form $\mathbf{K}R$ with a universal role R, it is simply replaced by R. In the following we assume that for any $\mathbf{K}R$, R is not universal.

Lemma 17. *Let Σ be a $\mathcal{SRIQ}$ knowledge base. For any epistemic role $R = \mathbf{K}P$ with $P \neq U$, and $x, y \in \Delta$ we have that $(x, y) \in R^{\mathcal{I},\mathcal{M}(\Sigma)}$ iff there are individuals $a, b \in N_I$ such that*

- $a^{\mathcal{I},\mathcal{M}(\Sigma)} = x$;
- $b^{\mathcal{I},\mathcal{M}(\Sigma)} = y$;
- $\Sigma_{\mathrm{UNA}} \models P(a, b)$.

Proof " $\Leftarrow$ "
By assumption we have that $\Sigma_{\mathrm{UNA}} \models P(a, b)$. Therefore, we have that $(x, y) \in P^{\mathcal{I}}$ for any interpretation $\mathcal{I} \in \mathcal{M}(\Sigma)$. Hence $(x, y) \in \mathbf{K}P$.
" $\Rightarrow$ "

Suppose there is no such $a, b \in N_I$. We distinguish two cases.

First assume there are a, b with $x = a^{\mathcal{I},\mathcal{M}(\Sigma)}$ and $y = b^{\mathcal{I},\mathcal{M}(\Sigma)}$ but $\Sigma_{\mathrm{UNA}} \not\models P(a, b)$. Then, there is an interpretation $\mathcal{I}'$ with $(a^{\mathcal{I}'}, b^{\mathcal{I}'}) \notin P^{\mathcal{I}'}$. Considering $\mathcal{I}'_\omega$, we can invoke Lemma 14 to ensure $\mathcal{I}'_\omega \models \Sigma_{\mathrm{UNA}}$ and by construction we also obtain $(a^{\mathcal{I}'_\omega}, b^{\mathcal{I}'_\omega}) \notin P^{\mathcal{I}'_\omega}$. Let $\varphi : \Delta^{\mathcal{I}'} \times \mathbb{N} \to \Delta$ be a bijection such that $\varphi(c^{\mathcal{I}'_\omega}) = c^{\mathcal{I}}$ for all $c \in N_I$. Such a φ exists, as $|\Delta^{\mathcal{I}'} \times \mathbb{N}| = |\Delta|$ and $\mathcal{I}'_\omega$ satisfies the unique name assumption. By Lemma 11, we get that $\varphi(\mathcal{I}'_\omega) \in \mathcal{M}(\Sigma)$. Moreover $(a^{\varphi(\mathcal{I}'_\omega)}, b^{\varphi(\mathcal{I}'_\omega)})) \notin P^{\varphi(\mathcal{I}'_\omega)}$. In particular,

$$(a, b) = (x, y) \notin \bigcap_{\mathcal{J}\in\mathcal{M}(\Sigma)} P^{\mathcal{J}}$$

which is a contradiction.

Second, assume at least one of x, y is anonymous. W.l.o.g. let x be anonymous, the other case follows by symmetry. Considering $\mathcal{I}_\omega$, we again have $\mathcal{I}_\omega \models \Sigma_{\mathrm{UNA}}$ by Lemma 14. By construction, $\langle x, 1\rangle$ is anonymous and $(\langle x, 1\rangle, \langle y, 0\rangle) \notin P^{\mathcal{I}_\omega}$. Let $\varphi : \Delta^{\mathcal{I}} \times \mathbb{N} \rightarrow \Delta$ be a bijection such that $\varphi(\langle x, 1\rangle) = x$ and $\varphi(\langle y, 0\rangle) = y$. Such a φ exists, since $|\Delta^{\mathcal{I}} \times \mathbb{N}| = |\Delta|$ and $\mathcal{I}_\omega$ satisfies the unique name assumption. By Lemma 11, we get that $\varphi(\mathcal{I}_\omega) \in \mathcal{M}(\Sigma)$. Moreover $(x^{\varphi(\mathcal{I}_\omega)}, y^{\varphi(\mathcal{I}_\omega)})) \notin P^{\varphi(\mathcal{I}_\omega)}$. In particular,

$$(x, y) \notin \bigcap_{\mathcal{J} \in \mathcal{M}(\Sigma)} P^{\mathcal{J}}$$

which again is a contradiction. □

In the following lemma, we show that the extension of a $\mathcal{SROIQK}$-concept and the extension of its translation obtained by applying Φ (see Defintion 9) agree under each model of the knowledge base.

Lemma 18. *Let Σ be a $\mathcal{SRIQ}$-knowledge base, x be an element of Δ, and C be a $\mathcal{SROIQK}$ concept. Then for any interpretation $\mathcal{I} \in \mathcal{M}(\Sigma)$, we have that $C^{\mathcal{I},\mathcal{M}(\Sigma)} = (\Phi(C))^{\mathcal{I},\mathcal{M}(\Sigma)}$.*

Proof. The proof is simply by induction on the structure of the formula. For the base case; C is an atomic concept, and the cases where $C = \top$ or $C = \bot$, the lemma follows immediately from the definition of Φ. For the cases, where $C = C_1 \sqcap C_2$, $C = C_1 \sqcup C_2$ or $C = \neg D$, it follows from the standard semantics and induction hypothesis. We focus on the rest of the cases in the following.

i. $C = \mathbf{K}D$ and $\Sigma_{\text{UNA}} \not\models D \equiv \top$:
By Lemma 15, $x \in (\mathbf{K}D)^{\mathcal{I},\mathcal{M}(\Sigma)}$ if and only if there is an $a \in N_I$ with $x = a^{\mathcal{I},\mathcal{M}(\Sigma)}$ and $\Sigma_{\text{UNA}} \models D(a)$. This is equivalent to $x \in \{a \in N_I \mid \Sigma \models D(a)\}^{\mathcal{I},\mathcal{M}(\Sigma)}$ and hence, by definition of Φ, to $x \in (\Phi(\mathbf{K}D))^{\mathcal{I},\mathcal{M}(\Sigma)}$.

ii. $C = \mathbf{K}D$ and $\Sigma_{\text{UNA}} \models D \equiv \top$:
Note that it trivially holds that if $x \in^{\mathcal{I},\mathcal{M}(\Sigma)}$ then $x \in (\Phi(C))^{\mathcal{I},\mathcal{M}(\Sigma)}$ as $\Phi(C) = \top$. Hence we just prove that whenever $x \in (\Phi(C))^{\mathcal{I},\mathcal{M}(\Sigma)}$ then $x \in C^{\mathcal{I},\mathcal{M}(\Sigma)}$ also. To contrary, suppose this is not the case i.e., $x \in (\Phi(C))^{\mathcal{I},\mathcal{M}(\Sigma)}$ but $x \notin C^{\mathcal{I},\mathcal{M}(\Sigma)}$. Hence, by definition, we get that

$$x \notin \bigcap_{\mathcal{J} \in \mathcal{M}(\Sigma)} D^{\mathcal{J}}$$

Therefore, there is an interpretation $\mathcal{I}' \in \mathcal{M}(\Sigma)$ such that $x \notin D^{\mathcal{I}'}$. Since $\mathcal{M}(\Sigma)$ is an epistemic interpretation, hence $\mathcal{I}' \in \mathcal{M}(\Sigma)$ respects the unique name assumption and therefore, $\mathcal{I}' \models \Sigma_{\text{UNA}}$ with $D^{\mathcal{I}'} \neq \Delta$. Hence $\Sigma_{\text{UNA}} \not\models D \equiv \top$, which is a contradiction.

iii. $C = \exists P.D$ and P is a simple role:
By semantics, $x \in (\exists P.D)^{\mathcal{I},\mathcal{M}(\Sigma)}$ if and only if there is $y \in \Delta$ such that $(x, y) \in P^{\mathcal{I},\mathcal{M}(\Sigma)}$ and $y \in D^{\mathcal{I},\mathcal{M}(\Sigma)}$, and therefore by induction, $y \in (\Phi(D))^{\mathcal{I},\mathcal{M}(\Sigma)}$. Hence it is equivalent to $x \in (\Phi(KD))^{\mathcal{I},\mathcal{M}(\Sigma)}$.

iv. $C = \exists \mathbf{K}P.D$:
$x \in (\exists \mathbf{K}P.D)^{\mathcal{I},\mathcal{M}(\Sigma)}$
$\Leftrightarrow$ there is $y \in \Delta$ such that $(x, y) \in (\mathbf{K}P)^{\mathcal{I},\mathcal{M}(\Sigma)}$ and $y \in D^{\mathcal{I},\mathcal{M}(\Sigma)}$
$\Leftrightarrow$ there is $y \in \Delta$ and by Lemma 15 there are $a, b \in N_I$ such that:
- $x = a^{\mathcal{I},\mathcal{M}(\Sigma)}$,
- $y = b^{\mathcal{I},\mathcal{M}(\Sigma)}$,
- $\Sigma_{\text{UNA}} \models P(a, b)$

and by induction hypothesis, $y \in \Phi(D)^{\mathcal{I},\mathcal{M}(\Sigma)}$. Note that x is related to y via P under every interpretation in $\mathcal{M}(\Sigma)$, hence by the rigid term assumption we get that $\Sigma \models \Phi(D)(b)$.

$\Leftrightarrow$ there is $y \in \Delta$ and by Lemma 15 there are $a, b \in N_I$ such that:
- $x = a^{\mathcal{I},\mathcal{M}(\Sigma)}$,
- $y = b^{\mathcal{I},\mathcal{M}(\Sigma)}$,

and $a \in \{c \in N_I \mid \exists d \in N_I : \Sigma \models P(a, b) \wedge \Sigma \models \Phi(D)(d)\}$
$\Leftrightarrow x \in \{c \in N_I \mid \exists d \in N_I : \Sigma \models P(a, b) \wedge \Sigma \models \Phi(D)(d)\}^{\mathcal{I},\mathcal{M}(\Sigma)}$
$\Leftrightarrow x \in [\Phi(\exists \mathbf{K}P.D)]^{\mathcal{I},\mathcal{M}(\Sigma)}$

v. The rest of the cases can be proved in a similar fashion.

In the following theorem we now present the main result of this work. We show that the problem of epistemic query answering formulated in $\mathcal{SROIQK}$ to a $\mathcal{SRIQ}$ knowledge base can be reduced to the problem of non-epistemic query answering.

Theorem 19. *For a $\mathcal{SRIQ}$ knowledge base Σ, $\mathcal{SROIQK}$-concepts C and D and an individual a the following hold:*

1. *$\Sigma \models\!\!\models C(a)$ exactly if $\Sigma_{\text{UNA}} \models \Phi(C)(a)$.*
2. *$\Sigma \models\!\!\models C \sqsubseteq D$ exactly if $\Sigma_{\text{UNA}} \models \Phi(C) \sqsubseteq \Phi(D)$.*

Proof. For the first case, we see that $\Sigma \models\!\!\models C(a)$ is equivalent to $a^{\mathcal{I},\mathcal{M}(\Sigma)} \in C^{\mathcal{I},\mathcal{M}(\Sigma)}$ which by Lemma 18 is the case exactly if $a^{\mathcal{I},\mathcal{M}(\Sigma)} \in \Phi(C)^{\mathcal{I},\mathcal{M}(\Sigma)}$ for all $\mathcal{I} \in \mathcal{M}(\Sigma)$. Since $\Phi(C)$ does not contain any **K**s, this is equivalent to $a^{\mathcal{I}} \in \Phi(C)^{\mathcal{I}}$ and hence to $\mathcal{I} \models \Phi(C)(a)$ for all $\mathcal{I} \in \mathcal{M}(\Sigma)$. Now we can invoke Fact 8 and Lemma 14 to see that this is the case if and only if $\Sigma_{\text{UNA}} \models \Phi(C)(a)$. The second case is proven in exactly the same fashion. □

Hence one can use standard DL-reasoners in order to answer epistemic queries. It can be seen from the definition of Φ that such answering of epistemic queries may require many sub-queries, hence involves many calls to the reasoners. Nevertheless, the number of calls to a reasoner is bounded by the size of N_I i.e., the number of individuals occurring in Σ and the number of **K**s occurring in the query.

4 Semantical Problems Caused by Nominals and the Universal Role

One of the basic assumptions we make regarding the epistemic interpretations is the *common domain assumption* as mentioned in Section 2.2. It basically has two parts: all the interpretations considered in an epistemic interpretation share the same fixed domain and the domain is infinite. However, there is no prima facie reason, why the domain that is described by a knowledge base should not be finite, yet finite models are excluded from the consideration entirely. While this is still tolerable for description logics up to $\mathcal{SRIQ}$ due to the fact that every finite model of a knowledge base gives rise to an infinite one that behaves the same (i.e. the two models cannot be distinguished by means of the underlying logic), as shown in Lemma 14. This situation changes once nominals or the universal role are allowed. In fact, the axioms $\top \sqsubseteq \{a, b, c\}$ or $\top \subseteq \leqslant 3U.\top$ have only models with at most three elements. Consequently, according to the prevailing epistemic semantics, these axioms are epistemically unsatisfiable. In general, the coincidence of $\models\!\!\models$ and $\models$ under the UNA which holds for non-epistemic KBs and axioms up to $\mathcal{SRIQ}$ does not hold any more, once nominals or the universal role come into play.

We believe that this phenomenon is not intended but rather a side effect of a semantics crafted for and probed against less expressive description logics, as it contradicts the intuition behind the **K** operator. A refinement of the semantics in order to ensure an intuitive behavior also in the presence of very expressive modeling features is subject of ongoing research.

5 A System

6 Conclusion and Related Work