# Distributed Geometric Distance Estimation in Ad Hoc Networks 

Sabrina Merkel, Sanaz Mostaghim, and Hartmut Schmeck<br>Karlsruhe Institute of Technology - Institut AIFB<br>76128 Karlsruhe, Germany<br>\{sabrina.merkel, sanaz.mostaghim, hartmut.schmeck\}@kit.edu<br>http://www.kit.edu


#### Abstract

Localization of nodes in ad hoc networks is an essential step in many applications. A major task when localizing nodes is to accurately estimate distances. So far, distance estimation is often based on counting the minimum number of nodes on the shortest routing path (hop count) and presuming a fixed width for one hop. This is prone to error as the length of one hop can vary significantly. The geometric distance estimation relies on the number of shared communication neighbors and applies geometric coherences to the network structure. In this paper, it is shown that the geometric approach is suitable to reliably estimate the distance between any two adjacent nodes in a network. Experiments reveal that the estimation has less relative percentage error compared to a hop based algorithm. Experiments are performed in networks with different node distributions to investigate the distributions' effect on the quality of the geometric approach.


Keywords: Ad Hoc networks, localization, distance estimation

## 1 Introduction

In many applications such as geographic monitoring, smart buildings, target tracking or disaster management, a large number of possibly mobile devices is utilized to accomplish a specific goal. In general, such devices consist of low power processors, have little memory and limited wireless communication range to exchange short messages with other devices. A network of such devices which has no fixed topology is called an ad hoc network or mobile ad hoc network (MANET) [1].
In such networks, adding a GPS-receiver to the devices might not always be desirable. Even though GPS meanwhile offers accuracy between 1 and 15 meters in horizontal positioning [2], it consumes a significant amount of power and is still quite expensive. Also, the GPS signal might not always be accessible, such as in an indoor or underwater scenario. Nevertheless, location-awareness plays an important role in many applications such as the allocation of event reporting in a monitoring sensor network [4], [5], [6], location dependent routing [7], [8], [9], [10], [11] assist group querying [12], pattern formation [13], [14]
and many more. For that reason, alternative localization techniques were proposed for ad hoc networks to derive the location of each device in the network in a self-organized, distributed manner (cf. [15], [16]).
Many of these algorithms rely on the estimation of the distance between each node in the network and a small number of so called anchor nodes which are assumed to know their coordinates either through a GPS-receiver or due to a priori configuration. There are two main ways to estimate distances, either by analyzing a received communication signal or based on hop counts. To determine the hop count, the anchor sends a message with value 0 to all its neighbors. Each node in the network takes the received message with minimal value, increments it by 1 , and forwards it accordingly. The width of one hop is then estimated as a function of the radius $r$, assuming equal or at least similar length for each hop. This is a daring assumption, because especially in sparse networks it is rarely the case.
Different from the existing approaches, the main idea of geometric distance estimation is based on estimating the distance between two adjacent nodes taking into account the individual local conditions. The goal of this paper is to transfer geometric coherences to the network structure and to use them for distance estimation. It is shown that the resulting distances are more accurate than distances derived by hop based approaches.
This paper is structured as follows. In section 2, the related work is summarized and the ad hoc network model is described briefly. In section 3, the geometric distance estimation algorithm is specified. Section 4 presents the experiments' settings and displays and discusses the results. Section 5 concludes the paper.

## 2 Basics

In this section related work for localization in ad hoc networks is presented and an overview of the network model is described.

### 2.1 Related Work

In the literature, a typical way for self-organized absolute positioning of nodes in a network is to use anchor nodes with known positions as a basis to derive coordinates for all other nodes in the network. Examples for such positioning algorithms are multilateration [21], triangulation [22], diffusion [23], [24] or bounding box [25], [26] approaches. As opposed to the diffusion mechanism, all other algorithms use distance estimation between each node and the anchors to calculate the nodes' coordinates.
There are several methods for estimating the distance between two nodes of an ad hoc network. The most commonly addressed way, is to use the strength of the radio frequency signal [3], [27], [28], [29]. Another method relying on the interpretation of the physical signal is called time-of-flight [30], [31] where an audio and infrared signal are emitted at the same time and the difference between the
arrival times is used to estimate the distance. Signal strength or pattern analysis "requires extensive pre-planning" [23] and for the time-of-flight technique complex hardware is required to be able to send and receive different signals and measure the time difference. To avoid these issues, several mathematical approaches are developed to computed distances in a distributed way only relying on local communication. To derive the distance communication hops between the node and the anchor are counted and multiplied with an estimated value for the width of one hop. The "DV-Hop propagation method" [3], [18], [19] estimates the width of one communication hop as the ratio of the physical distance between two anchor nodes and the number of hops that lie between them. In [17], the determined hop counts are averaged over a neighborhood and multiplied by the signal radius $r$, generally assuming $r$ as the length of each hop and then refining the situation of a node within its own hop. In [20], several fixed reduction rates depending on the density of nodes in the neighborhood are applied to $r$ to estimate the width of one hop. This improves the estimation of distances, especially in sparse networks, as it takes local information into account. One shortcoming is that the reduction rates and the density range they are applied for have to be chosen manually and beforehand. All of the afore mentioned methods estimate the distance across communication hops using one or several fixed estimates for the width of each hop.

### 2.2 Model

The applied model of an ad hoc network assumes randomly distributed devices on a two dimensional obstacle free plane. The devices do not have global knowledge of the network topology or their locations. Each device can communicate with adjacent devices, which are all devices in its neighborhood. The goal is for each device is to compute its distance to another node in the network using a decentralized algorithm only relying on local communication. The neighborhood of a device is defined as a physical neighborhood on the plane within a fixed distance $r$ from the device. The radius $r$ is assumed to be much smaller than the dimensions of the plane. All devices are assumed to have the same properties (homogeneous devices), except for anchor devices which posses knowledge of their own positions. Even though mobility is not regarded in this paper, the adjustment of the presented distance estimation algorithm to a mobile network is straightforward.

## 3 Geometric Distance Estimation

The basic idea of Geometric Distance Estimation is to approximately determine the common surface of two overlapping communication areas by the ratio of shared to total neighbors. Knowing the overlap surface $O$, the distance between the two communicating nodes can be derived. The distance can then be used as input for the localization algorithms presented in 2.1 to obtain coordinates for each device. In this section it is shown how to estimate the surface of the


Fig. 1. Two examples for adjacent nodes $i, j$ and their neighborhoods. Nodes with dotted lines belong to $N_{i}$, Grey filled nodes to $N_{j} . S_{i j}$ are nodes in the shaded area.
communication area overlap of two adjacent nodes and the necessary steps to derive an estimate for the distance between the two nodes. The requirements for the geometric distance estimation algorithm are that each node knows all of its neighbors and can communicate with them and that the communication radius $r$ is identical and known to all devices. For a node $i$ to estimate the portion of its communication area which it has in common with an adjacent node $j$ the neighbors of node $i$ have are distinguished with respect to $j$ as follows:

Definition 31 (Classification of Neighbors) Let $i$, $j$ be two adjacent nodes and $N_{i}, N_{j}$ the nodes situated in the neighborhood of $i$ and $j$ respectively. The neighbors of $i$ can now be categorized with respect to $j$ as:

$$
\begin{aligned}
& \text { shared neighbors: } S_{i j}:=\left\{n \mid n \in\left(N_{i} \cap N_{j}\right)\right\} \\
& \text { individual neighbors: } I_{i j}=\left\{n \mid n \in\left(N_{i} \backslash S_{i j}\right)\right\}
\end{aligned}
$$

Figure 1 shows two examples for adjacent nodes $i$ and $j$ and the corresponding classification of their neighbors.

The network structure of two adjacent nodes and their communication areas can be mapped to the geometrical shape of two overlapping cirles. The problem to determine the distance between the adjacent nodes is hence transfered to computing the distance between the corresponding circles' centers. The ratio of shared $S_{i j}$ to total neighbors $N_{i}$ of a node $i$ might deliver a good estimate for the ratio of overlapping to total circular surface area. Assuming this correlation holds, the surface of the overlapping area $O$ can be estimated from the perspective of node $i$ as $O \approx \pi r^{2} \cdot \frac{\left|S_{i j}\right|}{\left|N_{i}\right|}$.

The circles' cut surface $O$ has the shape of a concave lens or a mirrored circular segment with surface $A$ (cf. Figure 2), with:

$$
\begin{equation*}
A \approx 0.5 \cdot \pi r^{2} \cdot \frac{\left|S_{i j}\right|}{\left|N_{i}\right|} \tag{1}
\end{equation*}
$$

When two circles of the same surface overlap, the cut's surface $O$ should be inverse proportional to the distance $d$ between the circles' centers. Standard


Fig. 2. Geometric characteristics of two overlapping circles with the overlap surface $O$ (blue filled area) and the circular segment surface $A$ (dotted area).

Height Ratio in Relation to Segment Ratio


Fig. 3. Relation of $\theta$ to $\Delta$ and the approximated third-degree polynomial function $f$.
equation (2) describes the segment surface $A$ from known radius $r$ and segment height $h$.

$$
\begin{equation*}
A=r^{2} \arccos \left(1-\frac{h}{r}\right)-\sqrt{2 r h-h^{2}}(r-h) \tag{2}
\end{equation*}
$$

With known $A$ and $r$ one could try to derive the value of $h$ from equation 2. The segment height $h$ can be mapped to the distance $d$ between the circles' centers with known $r$. The distance between the center of the circle and the chord is equal to $r-h$. Therefore, the distance between the two centers can be obtained by:

$$
\begin{equation*}
d=2 \cdot(r-h) \tag{3}
\end{equation*}
$$

Resolving Equation (2) to $h$ is not feasible and as Equation 2 depends on $h$ and $r$ there is no 2 -dimensional representation that could be approximated by the usage of regression. Nevertheless, the following considerations help to solve this problem.
The height $h$ of a segment can be described as a ratio $\theta$ of the circle's radius $r$ and the segment area $A$ is a portion of half the circle's surface:

$$
\begin{equation*}
\theta=\frac{h}{r} \quad(4) \quad \Delta=\frac{A}{0.5 \cdot \pi r^{2}} \tag{4}
\end{equation*}
$$

In the following we show that $\Delta$ and $\theta$ are independent of $r$ with the result that the relationship between $\Delta$ and $\theta$ can be approximated using regression.

The standard equations 7 and 6 descirbe $A$ and $h$ depending on $r$ and angle $\alpha$ (cf. Figure 2).

$$
\begin{equation*}
A=\frac{r^{2}}{2} \cdot(\alpha-\sin (\alpha)) \quad(6) \quad h=r \cdot\left(1-\cos \left(\frac{\alpha}{2}\right)\right) \tag{6}
\end{equation*}
$$

Substitution $A$ and $h$ by rearranging Equations 5 and 4, it becomes apparent that $\Delta$ and $\theta$ are only dependent on $\alpha$, which has a fixed value range, not on $r$.

The relation of $\Delta$ and $\theta$ is now independent of $r$ and can be approximated using regression. Figure 3 shows data points (grey line) and the approximated third-degree polynomial function $f: \Delta \rightarrow \theta$ (dotted line) derived through polynomial regression. Apparently, $f$ is an almost perfect approximation of the relationship between $\Delta$ and $\theta$.

From the approximated function $f$, an estimate for the segment height $h$ and, thus, the distance $d$ can be calculated with known $\Delta$ :

$$
\begin{equation*}
d=2 r(1-2 \cdot f(\Delta)) \tag{8}
\end{equation*}
$$

As stated before, $A$ can be estimated from the relation between shared neighbors $S_{i j}$ to total neigbhors $N_{i}$ which can be computed locally using Equation 1.

Putting it all together, Equation (9) calculates the distance estimate $\hat{d}_{i j}$ for node $i$ to its adjacent neighbor $j$, given the number of shared neighbors $\left|S_{i j}\right|$, total neighbors $\left|N_{i}\right|$ and $r$.

$$
\begin{equation*}
\left.\left.\hat{d}_{i j}=r \cdot\left(a \cdot\left(\frac{\left|S_{i j}\right|}{\left|N_{i}\right|}\right)^{3}+b \cdot\left(\frac{\left|S_{i j}\right|}{\left|N_{i}\right|}\right)^{2}+c \cdot\left(\frac{\left|S_{i j}\right|}{\left|N_{i}\right|}\right)+e\right)\right)\right) \tag{9}
\end{equation*}
$$

Using regression to determine the polynomial $f$ and further computations one can estimate the coefficients as:

$$
a=3.90 \quad b=-4.16 \quad c=3.04 \quad e=0.04
$$

### 3.1 Evaluation of Geometric Distance Estimation

There are two influences for the accuracy of the geometric distance approach. Firstly, the approximation of $A$ using Equation 1 depends on the distribution of neighbors in the communication area as well as the neighborhood size $N_{i}$, secondly, the approximation of function $f$ using polynomial regression is a source of error.
The assumption underlying the geometric distance estimation approach is that the number of nodes within an area of the environment can be mapped to the size of this area. This is a critical assumption when the distribution of nodes is imbalanced. As a result the ratio of shared to individual neighbors might not reflect the relation of overlapping to total circular area anymore. Figure 1 (b) illustrates this effect. The impact of the nodes' distribution is assessed in the experiments shown in section 4 . The neighborhood size $N_{i}$ determines the possible precision for estimating $\Delta$. There are $\left|N_{i}\right|+1$ possible estimates for the ratio of segment surface area to total area $\Delta$. The margin between these values


Fig. 4. Approximation error of function $f$.
is $\frac{1}{\left|N_{i}\right|}$. The resulting possible absolute error for the estimation of $\Delta$ lies within the interval $\left[0, \frac{1}{\left|N_{i}\right|}\right)$. From Equation 8 and 9 the maximum absolute distance estimation error induced by a small neighborhood size can be calculated as $\epsilon \in[0,(28 a+12 b+4 c) r)$ with $\left|N_{i}\right|=1$ and $\Delta \rightarrow 1$.
The other source of error is the approximation of function $f$. Figure 4 shows the deviation between the approximation $f(\Delta)$ and the correspoinding calculated values of $\theta$ for different values of $\theta$. As Figure 4 indicates, the approximation error of function $f$ is at most of 0.04 . Which leads to a maximum absolute distance estimation error of $0.16 r$. The acutal error depends on the ratio of height $h$ to radius $r$ and, as the height is coupled with the distance $d$, it follows, that estimating the same distance with different radii $r$ can lead to different estimation errors.

### 3.2 Distributed Geometric Distance Estimation Algorithm for Ad Hoc Networks

In principle, the distance estimate $\hat{d}_{i j}$ can range between 0 and $r$ as the centers of two overlapping circles have a maximum distance of $2 r$. This ignores the fact, that adjacent nodes can have a maximum distance of $r$ to be able to communicate. Therefore, in the network scenario $\hat{d}_{i j}$ can be restricted to a maximum value of $r$. This corresponds to a limited height $h \in[0.5 r, r]$ and, thus, the approximation error of function $f$ is limited to the section highlighted in grey in Figure 4.
As neighborhoods of $i$ and $j, N_{i}$ and $N_{j}$, commonly differ in size (cf. Figure 1 for an example), the estimates node $i$ and node $j$ calculate for their distance can vary as well. An improved approximation can be obtained when node $i$ and node $j$ exchange their estimates via communication and calculate the average of $\hat{d}_{i j}$ and $\hat{d}_{j i}$.

This leads to the following algorithm computed by node $i$ to estimate its distance to the adjacent node $j$ using the geometric distance estimation approach:

```
Algorithm 1 CalcDistToNeighbor(i, j)
    // Computing the distance between \(i\) and a neighbor \(j\)
    Input: node \(i\) and node \(j\)
    Output: estimated distance \(\hat{d}_{i j}\)
        \(N_{i}=\) set of neighbors nodes
        Ask neighbor \(j\) to send its set of neighbors \(N_{j}\)
        Compute the shared neighbors \(S_{i j}\)
        Let \(x:=\frac{\left|S_{i j}\right|}{\left|N_{i}\right|}\)
        \(\hat{d}_{i j}=r \cdot\left(3.90 \cdot x^{3}-4.16 \cdot x^{2}+3.04 \cdot x+0.04\right)\)
        Limitation: If \(\left(\hat{d}_{i j}>r\right)\) Then \(\hat{d}_{i j}=r\)
        Averaging: Ask \(j\) for \(\hat{d}_{j i}\) and compute \(\hat{d}_{i j}=0.5 \cdot\left(\hat{d}_{i j}+\hat{d}_{j i}\right)\)
```

To transfer the presented concept to a long range distance estimation between a node $i$ and an anchor node $a$, all distances along the shortest path between both nodes are aggregated. The assumption is that all nodes in the network estimate their distance to the anchor node $a$, which is the case for all eligible localization algorithms (cf Section 2.1). To estimate the distance between node $i$ and an anchor node $a$ the following algorithm is computed on node $i$ :

```
Algorithm 2 CalcDistToAnchor(i, a)
// Computing the distance between \(i\) and an anchor \(a\)
Input: node \(i\) and node \(a\)
Output: estimated distance \(\hat{d}_{i a}\)
1: \(N_{i}=\) set of neighbors of node \(i\)
2: If anchor \(\left(a \in N_{i}\right)\) Then \(\hat{d}_{i a}=\) CalcDistToNeighbor(i, a)
3: Else search for neighbor \(k\) closest to \(a\) :
            For \(j=1\) To \(\left|N_{i}\right|\)
                \(k=\operatorname{argmin}\left(\hat{d}_{j a}=\operatorname{CalcDistToAnchor}(\mathrm{j}, \mathrm{a}), \mathrm{j}\right)\)
            End For
    4: Compute distance to \(k: \hat{d}_{i k}=\) CalcDistToNeighbor \((\mathrm{i}, \mathrm{k})\)
            Aggregate distances: \(\hat{d}_{i a}=\hat{d}_{i k}+\hat{d}_{k a}\)
    End If
```

For comparison, in [21], the distance $\hat{d}_{i a}$ between a node $i$ and the anchor $a$ is estimated as:

$$
\begin{equation*}
\hat{d}_{i a}=\frac{\sum_{j \in N_{i}} h_{j a}+h_{i a}}{\left|N_{i}\right|+1}-0.5 \times r \tag{10}
\end{equation*}
$$

With $N_{i}$ being all neighbors of node $i$ and $h_{i a}$ denoting the hop count of node $i$ to the anchor $a$.

Note that in both algorithms each node's calculation depends on other nodes' results. Therefore, the algorithm has to be executed iteratively before a stable estimate is achieved. The necessary number of executions is subject to the neighborhood size and the number of nodes that lie on the shortest path between $i$ and $a$. In mobile networks the algorithm can be executed repeatedly to dynamically compute the distance estimate considering changes in the locations of node $i$ or $a$ respectively.

## 4 Experiments

Geometric distance estimation relies on the idea that the ratio of shared to total neighbors delivers a sufficiently precise estimate for the ratio of overlapping to total surface of the communication area. In this section experiments are presented to evaluate whether this basic assumption holds for a variety of network topologies. The quality of the derived distance estimates are evaluated for three different network scenarios. The second part of the experiments investigates the usage of the geometric distance estimation approach to estimate distances to anchor nodes. The results are then compared to distances derived by a hop count based approach as described in 2.1.
For the experiments a 2-dimensional square environment of size $1.0 \times 1.0$ units containing 1000 nodes is considered. The neighborhood size as well as the physical distribution of nodes is expected to be an influencing factor for the quality of the estimate. Therefore, different scenarios for the nodes' distribution across the environment are considered. Two randomly distributed networks are investigated using a uniform random distribution in Scenario 1 (cf. Figure 5(c)) and a Gaussian random distribution in Scenario 2 (cf. Figure 5(b)). Figure 5(c) displays Scenario 3, where the nodes are positioned evenly in a grid-like shape. Applying a uniform random distribution to determine positions results in a balanced distribution of neighbors across the communication surface area similar to Figure 1(a), whereas the Gaussian random distribution distorts the balance in direction of the environment's center (cf. Figure 1(b) for an example). Evenly distributed networks have the characteristic that all inner nodes (nodes that have a minimum distance of $r$ to the border of the environment) have the same neighborhood size. These different scenarios were selected to investigate the influence of node distribution of neighbors in the communication area. Besides the distribution of nodes, different values for the communication radius $r$ were tested to vary the neighborhood size which was also identified to be a potential source of error (cf. Section 3.1).

### 4.1 Distance estimation between neighbors

In the first set of experiments, every node estimates its distance to all adjacent nodes, i.e. all nodes within communication range, using the geometric distance estimation approach. Due to lack of comparable algorithms (the other presented approaches only estimate distances that exceed one hop), the average distance


Fig. 5. Positioning according to a uniform random distribution (a), a Gaussian random distribution (b), and evenly distributed nodes (c).


Fig. 6. Average distances between adjacent nodes in networks with different distributions depending on the communication radius $r$.
between adjacent nodes in the considered scenarios is taken as reference. Figure 6 shows the average distances for each experiment setting. The step-like incline in evenly distributed networks is due to the symmetric arrangement of nodes. To evaluate the quality of the estimates, the mean absolute percentage error (MAPE) is calculated as $\operatorname{MAPE}\left(\hat{d}_{i j}\right)=\frac{\left|d_{i j}-\hat{d}_{i j}\right|}{d_{i j}}$, where $d_{i j}$ denotes the euclidean distance between a node $i$ and its neighbor $j$ and $\hat{d}_{i j}$ denotes the estimate of that distance. The MAPE gives information about the relative deviation of the estimate with respect to the real distance. As nodes near the border of the environment have a circumcised communication area, all experiments were repeated using only inner nodes in order to illustrate the influence of border nodes on the network's average estimation error.

The results for Scenario 1 are shown in Figure 7(a). The geometric distance estimation delivers estimation results ranging between $40 \%$ up to approximately $15 \%$ ( $10 \%$ for inner nodes) deviation from the real distance which is consistently less error-prone than estimating the distance using the average of the network. The results indicate, that the geometric distance estimation approach delivers reliable estimates for distances between adjacent nodes at least in networks with uniformly randomly distributed nodes. An observation that can be made is the improvement of estimation quality with increasing communication radius $r$. This can be explained by the entailed growth of the number of neighbors and, thus,


Fig. 7. MAPE using the geometric approach (Geo) compared to the error when using the average distance as an estimate (Simple).
the increase in precision.
Figure 7(b) shows the MAPE for distance estimation between any two adjacent nodes in a Gaussian random distributed network. Contrary to what one might intuitively expect, the geometrical estimation performs even better as in uniformly random distributed networks despite the imbalanced distribution of nodes. The reason lies in averaging the estimates of both involved nodes. An unbalanced distribution of nodes leads to an overestimation in one node and an underestimation in the other node which may, under certain circumstances, provide a good estimate on average. Another factor for the less error-prone estimates in the Gaussian distributed network is the larger average neighborhood size due to the concentration of nodes in the center of the environment.

For scenario 2, it is further noticeable, that the percentage error does not decrease continuously with rising radius $r$, which seemed to be the case for uniformly random distributed networks. Instead, the curve has a convex shape. This is due to the approximation error of $f$. As stated before, the estimation error induced by approximating the function $f$ is dependent on $\theta$, i.e. the ratio of height $h$ to radius $r$. For all experiments $\theta$ ranges between ( $0.61,0.69$ ), thus the closest zero-error point $\theta *$ lies approximately at $\theta *=0.745$ (cf. Figure 4 ). Figure 8 shows the average percentage deviation for all considered node distributions and radii
from this zero-error-point. The experiments with Gaussian distributed nodes diverge stronger with increasing radius than the experiments with uniformly random distributed nodes, which explains the convex behavior of the MAPE curve.


Fig. 8. Percentage deviation between $\theta$ and $\theta *$, i.e. the value for $\theta$ where $f$ has zero approximation error.


Fig. 9. Sample standard deviation of distance estimation between neighbors in the three considered scenarios.

Figure 7(c) shows the results for Scenario 3. Intuitively one would expect a similar MAPE as in uniformly random distributed networks, as the distribution of nodes is very balanced in both scenarios. Nevertheless, this does not appear to be the case at first sight, but when looking at the trendline (black dotted line) the behavior is quite similar. The oscillating error can be explained by the step-like increase of the average distance $d$ (cf. Figure 6) in combination with the afore mentioned distance dependent error of the approximated function $f$.

Figure 9 illustrates the sample standard deviation for the previously presented experiments. It shows that the standard deviation is relatively small compared to the estimates using the average distance. This further substantiates the observation that the geometric concept is successfully transferred to the network topology delivering reliable estimates for each regarded distance estimation and not only on average for the whole network.

### 4.2 Distance estimation to anchor nodes

The second set of experiments has the objective to evaluate the geometric distance estimation concept for the estimation of distances to anchor nodes. Therefore, an anchor node is randomly chosen in each experiment iteration and all other nodes estimate their distance to this anchor node according to Algorithm 2 (cf. Section 3.2). For comparison, the hop count based distance estimation described in [17] is used. This method has been successfully used for localization in [21] and does not require more than one anchor node for distance estimation as opposed to the DV-hop propagation model in [18].


Fig. 10. MAPE for geometric versus traditional approach on long distance estimation including standard sample deviation.

Figure 10(a) shows the MAPE for Scenario 1, using the uniform random distribution for node positioning. Figure 10(b) for Scenario 2, the Gaussian randomly distributed network and Figure 10(c) for Scenario 3, with evenly distributed nodes. It can be observed that the geometric distance estimation approach leads to less error-prone estimates than the hop count based estimation for all considered distributions and radii. Furthermore, it should be noted that even the sample standard deviation is many times less or equal to the MAPE of hop count based estimates. This confirms that the geometric distance estimation approach is a consistent improvement in distance estimation for all considered ad hoc network scenarios and radii.

## 5 Conclusion and Future Work

This paper presents an approach for estimating distances in an ad hoc network. The approach relies on the ratio of shared to total neighbors and applies geometric coherences to the network structure. Three sources for error in the geometric distance estimation approach were identified and, where possible, quantified. Experiments were conducted to investigate the absolute percentage error of the distance estimates in three different network scenarios: uniformly random, Gaussian random, and evenly distributed nodes. The results were compared to a hop
count based estimation approach, showing that the geometric distance estimation reliably delivers more precise estimates. This observation was consistent for all investigated communication radii and node distribution scenarios. Furthermore, even the sample standard deviation for geometric distance estimation is close to the average percentage error of the hop count based approach and lies below it for some considered experiment settings.
In future work, the geometric distance estimation method is to be investigated for the usage in localization algorithms described in section 2.1. We expect to improve the accuracy of the established coordinate system with the geometric distance estimation as a great part of the error in finding coordinates is due to inaccuracy in distance estimation. Besides, the robustness of the algorithm is to be tested under mobile conditions.

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